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**Effect of Aggregate on Shrink-  
age of Concrete and Hypothesis  
Concerning Shrinkage**

**BY**

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# Effect of Aggregate on Shrinkage of Concrete and a Hypothesis Concerning Shrinkage\*

By GERALD PICKETT†

## SYNOPSIS

A theoretical formula is derived for effect of aggregate on shrinkage of concrete during drying. Experiments designed to test the validity of the formula are reported.

In addition to indicating the validity of the formula, the data give the following indications: (1) First shrinkage is greater than any subsequent expansion or shrinkage resulting from moisture change. (2) At a given aggregate content the shrinkage is approximately proportional to water-cement ratio. (3) After first shrinkage, subsequent volume changes are approximately independent of water-cement ratio. (4) When shrinkages of specimens of the higher water-cement ratio are plotted against the square root of period of drying, the shapes of the curves for second shrinkage are appreciably different from those for first shrinkage in that they have considerable curvature near the origin. An explanation of these effects is given.

## INTRODUCTION

A number of years ago, while at the Portland Cement Assn., the author arrived at a theoretical formula for effect of aggregate on shrinkage of concrete or mortar during drying. Experiments designed to test the validity of the formula gave results that were in fair agreement with the formula. However, certain factors in the formula which should depend on properties of the paste varied with conditions of drying and therefore led to the conclusion that the hydrated paste did not always have the same properties. The purpose of this paper is to present the theoretical formula, experimental results that were obtained, and speculations in regard to the paste that resulted from a study of the data.

## DERIVATION OF FORMULA

In deriving the formula, consideration is first given to effect on shrinkage of one small, spherical particle of aggregate in a large body of concrete, the surrounding concrete considered to be a homogeneous material. This approach is similar to that of Guth<sup>1</sup> and Dewey,<sup>2</sup> who were concerned with the effect of fillers on elastic properties. The restraining effect of aggregate

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on shrinkage of concrete was pointed out by Carlson.<sup>3</sup> On the assumption that both the particle and the rest of the body are elastic, an expression is derived for reduction in over-all shrinkage of the body due to the one small, nonshrinking particle. This provides a formula for the effect of adding each subsequent particle if the body including all particles added previously is assumed to be homogeneous. This formula is then expressed in differential equation form and an integration made to obtain the final formula.

It will be expedient to consider that the small, spherical particle is at the center of the body of concrete which is also a sphere. If the particle is small compared to the shortest distance from it to the concrete surface, no great error will be introduced by treating the concrete as spherical with a radius equal to that distance. The restraint of the small sphere as the large sphere tends to shrink will cause the following stresses in the large sphere.<sup>4</sup>

$$\sigma_r = -\frac{pa^3}{r^3} \frac{b^3 - r^3}{b^3 - a^3} \dots\dots\dots(1)$$

$$\sigma_t = \frac{pa^3}{2r^3} \frac{b^3 + 2r^3}{b^3 - a^3} \dots\dots\dots(2)$$

where  $\sigma_r$  = normal stress in the radial direction

$\sigma_t$  = either of two normal stresses perpendicular to the radius

$r$  = radial coordinate

$a$  = radius of inner sphere

$b$  = radius of outer sphere

$p$  = unit pressure between inner and outer spheres

Under these conditions of spherical symmetry, radial displacement  $\delta$  of any point in the outer sphere caused by the restraint of the inner sphere, and referred to the unrestrained position, is

$$\delta = \frac{r}{E} \left[ (1 - \mu) \sigma_t - \mu \sigma_r \right] \dots\dots\dots(3)$$

where  $E$  and  $\mu$  are Young's modulus and Poisson's ratio, respectively, for the outer sphere.

From Eq. (1), (2), and (3)

$$\delta = \frac{pa^3}{Er^2} \left[ \frac{1 - \mu}{2} \frac{b^3 + 2r^3}{b^3 - a^3} + \mu \frac{b^3 - r^3}{b^3 - a^3} \right] \dots\dots\dots(4)$$

The restraint of the inner sphere has reduced the volume shrinkage of the total body by the amount

$$4\pi b^2 \delta \Big|_{r=b} = \frac{3p\Delta V}{E} \left( \frac{1 - \mu}{2} \right) \frac{3b^3}{b^3 - a^3} \dots\dots\dots(5)$$

where  $\Delta V = 4/3 \pi a^3$  is the volume of the small sphere.

If the restraint had not been present, the body would have reduced in volume by  $3SV$ , where  $V$  is its total volume and  $S$  is the unit linear shrinkage. The reduction in volume shrinkage will therefore be designated as  $-3\Delta SV$ , or

$$-3\Delta SV = \frac{3p\Delta V}{E} \left( \frac{1 - \mu}{2} \right) \frac{3b^3}{b^3 - a^3} \dots\dots\dots(6)$$

Another expression containing the pressure  $p$  will be found by considering the compressibility of the restraining particle. Reduction in volume of the



particle caused by pressure  $p$  on it will be equal to the reduced space available to it within the larger body, or

$$\frac{3(1 - 2\mu_g)p \Delta V}{E_g} = 3S \Delta V - 4\pi a^2 \delta \Big|_{r=a} \dots\dots\dots (7)$$

where  $E_g$  and  $\mu_g$  are the elastic constants of the particle and  $\delta$  is given by Eq. (4).

Eliminating  $p$  between Eq. (6) and (7) and setting  $b/a = \infty$  gives

$$-\Delta S V = \alpha S \Delta V \dots\dots\dots (8)$$

where

$$\alpha = \frac{3(1 - \mu)}{1 + \mu + 2(1 - 2\mu_g)E/E_g} \dots\dots\dots (9)$$

Setting  $b/a = \infty$  will introduce an error especially for particles close to the surface. However, it is believed that this error is not relatively as important as others entering this derivation.

Let volume of aggregate per unit volume of mix be  $g$ ; then the increase in  $g$  due to the addition of one particle of volume  $\Delta V$  to the mixture will be

$$\Delta g = \frac{gV + \Delta V}{V + \Delta V} - g = (1 - g) \frac{\Delta V}{V + \Delta V} \dots\dots\dots (10)$$

From Eq. (8) and (10)

$$\frac{\Delta S}{S} = - \frac{\alpha \Delta g}{1 - g} \frac{V + \Delta V}{V} \dots\dots\dots (11)$$

or, in differential form,

$$\frac{dS}{S} = - \frac{\alpha dg}{1 - g} \dots\dots\dots (12)$$

The factor  $\alpha$  is probably a function of  $g$  since the elastic constants of the mixture,  $E$  and  $\mu$ , may depend on  $g$ . But if  $\alpha$  may be considered to be independent of  $g$ , then Eq. (12) integrates to

$$S = S_0 (1 - g)^\alpha \dots\dots\dots (13)$$

where  $S_0$  is the shrinkage that would occur if no aggregate were present. For later use this equation may be written in the form

$$\log \frac{S_0}{S} = \alpha \log \frac{1}{1 - g} \dots\dots\dots (14)$$

## TESTS

To test the validity of the formula,  $1 \times 7/8 \times 11 1/4$ -in. prisms were prepared with various percentages of aggregate ranging from 0 percent to about 70 percent by volume. Three different types of aggregate (pulverized silica, standard Ottawa sand, and graded Elgin sand) were used to determine whether size and gradation of aggregates would also be an appreciable factor. Two cements, a high-early-strength and a normal, and two water-cement ratios were used to determine in what way the effect of aggregate might be influenced by type of cement and water-cement ratio.



**TABLE 1—GENERAL OUTLINE OF CONDITIONS IN STUDY\***

Cements used	Aggregates used	Percent aggregate by absolute volume†	W/C by weight
High-early-strength	Silica flour	0	0.5
	Standard Ottawa sand	5	
Laboratory mix of four brands of Type I	Standard Ottawa sand	15	0.35
	Graded Elgin sand	30	
		50	
		65	

\*Specimens were sealed in steel molds  $\frac{7}{8} \times 1 \times 11\frac{1}{4}$  in., that were stood on end for 2 hr during setting of mortar. Each mold was turned end-for-end every 5 min. Curing was for 7 days under water at 78 F. Drying was for 224 days or longer at 50 percent relative humidity, 76 F and the wetting period was 84 days under water at 74 F. Two specimens were made of each combination, a total of 72.

†As will be noted later, there were slight deviations from these values.

Later it was decided to investigate reversibility of volume changes of these specimens. For this purpose specimens were alternately submerged in water and dried in air. Each drying was at 50 percent relative humidity for at least 224 days and each period of wetting was 84 days. This work was begun in January, 1942, and continued for about 2 years.

Table 1 gives the general outline of conditions covered in the study.

Mixes containing up to 5 percent aggregate were too wet and those with

50 percent or more were too dry for preparation of reasonably homogeneous specimens. Repeated reversal of the position of the molds during setting tended to offset some effects of bleeding, but many of the wet mixes were blemished because of the combined effects of bleeding, shrinkage in absolute volume, and periodic turning of the molds. Some dry mixes had a high percentage of air voids though in most cases percentage of air was kept low by vigorous tamping. The wide range in plastic properties of the mixes may account for some of the nonuniformities in results.

Shrinkage during drying and expansion during wetting for the specimens made with high-early-strength cement are shown graphically in Fig. 1 and 2. Similar results were obtained with normal cement but are not shown.

Final shrinkage indicated by each curve was estimated. These results are given in Table 2.

The quantities  $\log S_0/S$  and  $\log 1/(1 - g)$  were computed from Table 2 and plotted in Fig. 3. According to Eq. (14) the data should be represented by a straight line passing through the origin. The data for  $W/C = 0.35$  are represented fairly well by such a straight line with a slope  $\alpha$  equal to 1.7. Various combinations of the elastic constants in Eq. (9) would make  $\alpha = 1.7$ . A reasonable assumption that would give this value is:  $\mu = 0.2$ ,  $\mu_0 = 0.25$ ,  $E/E_0 = 0.21$ .

Data for  $W/C = 0.50$  are not represented as well by a straight line. The line shown has a slope of 1.7. Points which lie farthest from the line are for mixes that were stiff and when cast contained a considerable percentage of air voids. If these points are neglected, the agreement is excellent.

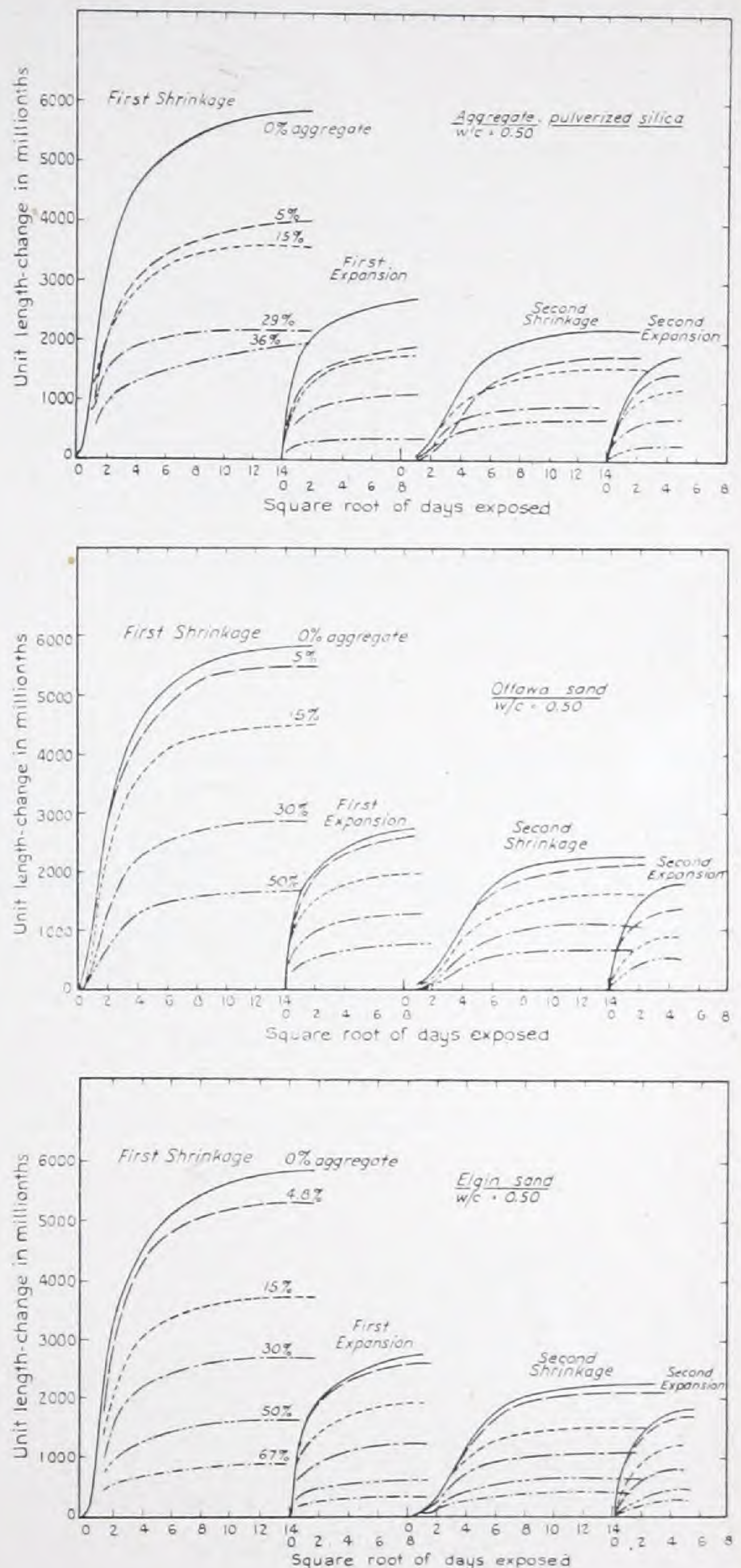
On the basis of data obtained it is concluded that the derived formula gives a good representation of the effect of aggregate on ultimate shrinkage due to change in moisture.



### Other ideas suggested by the data

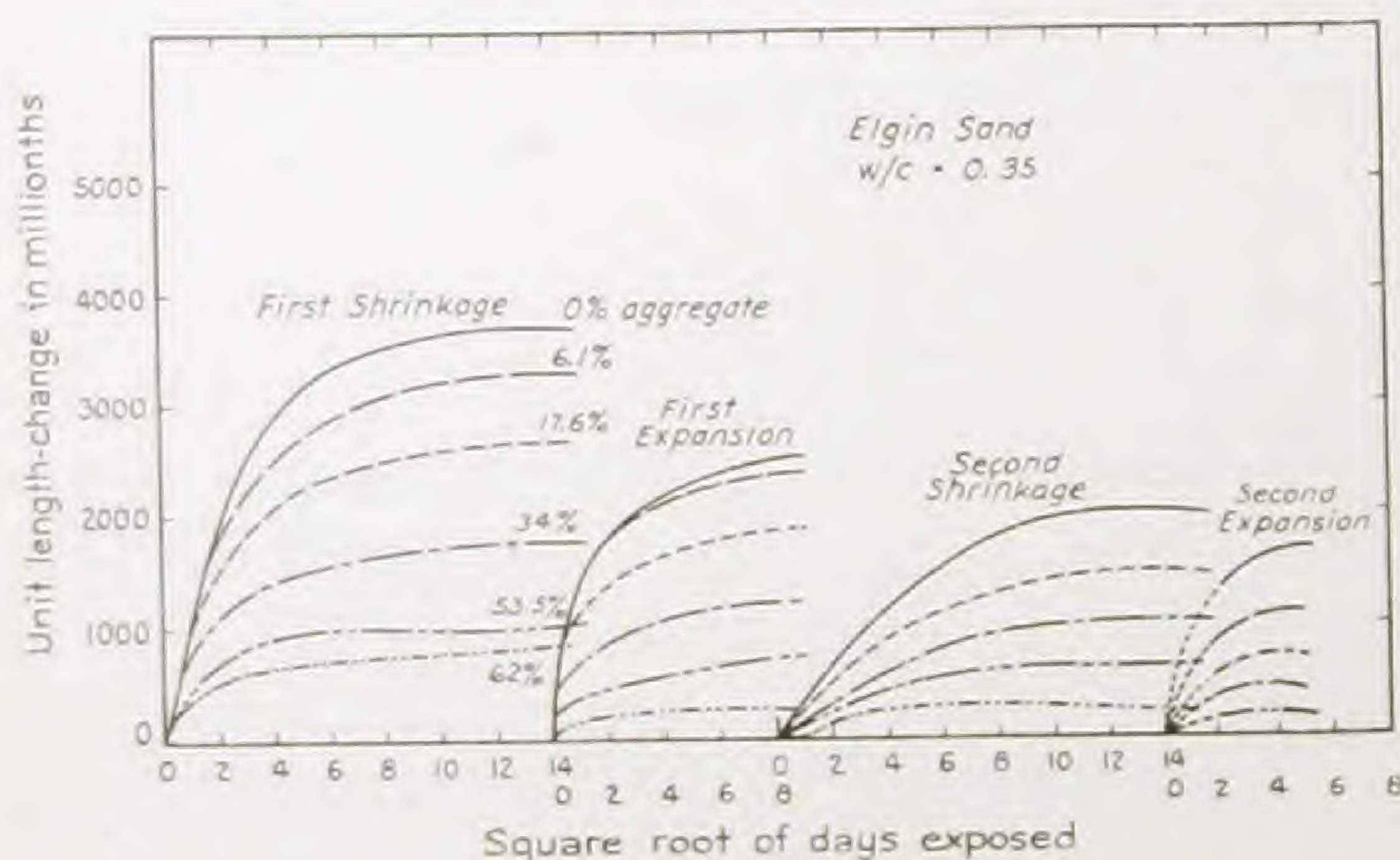
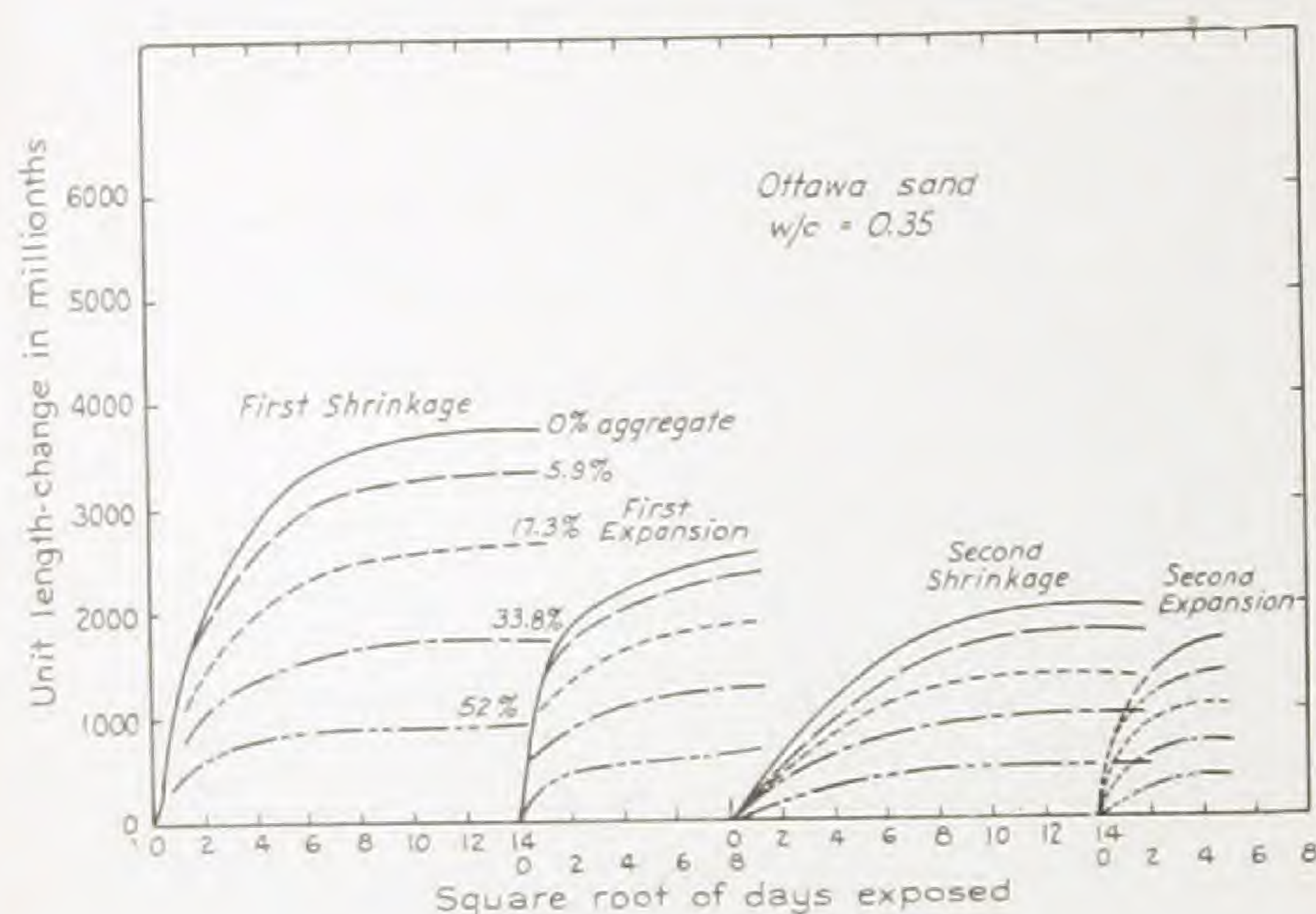
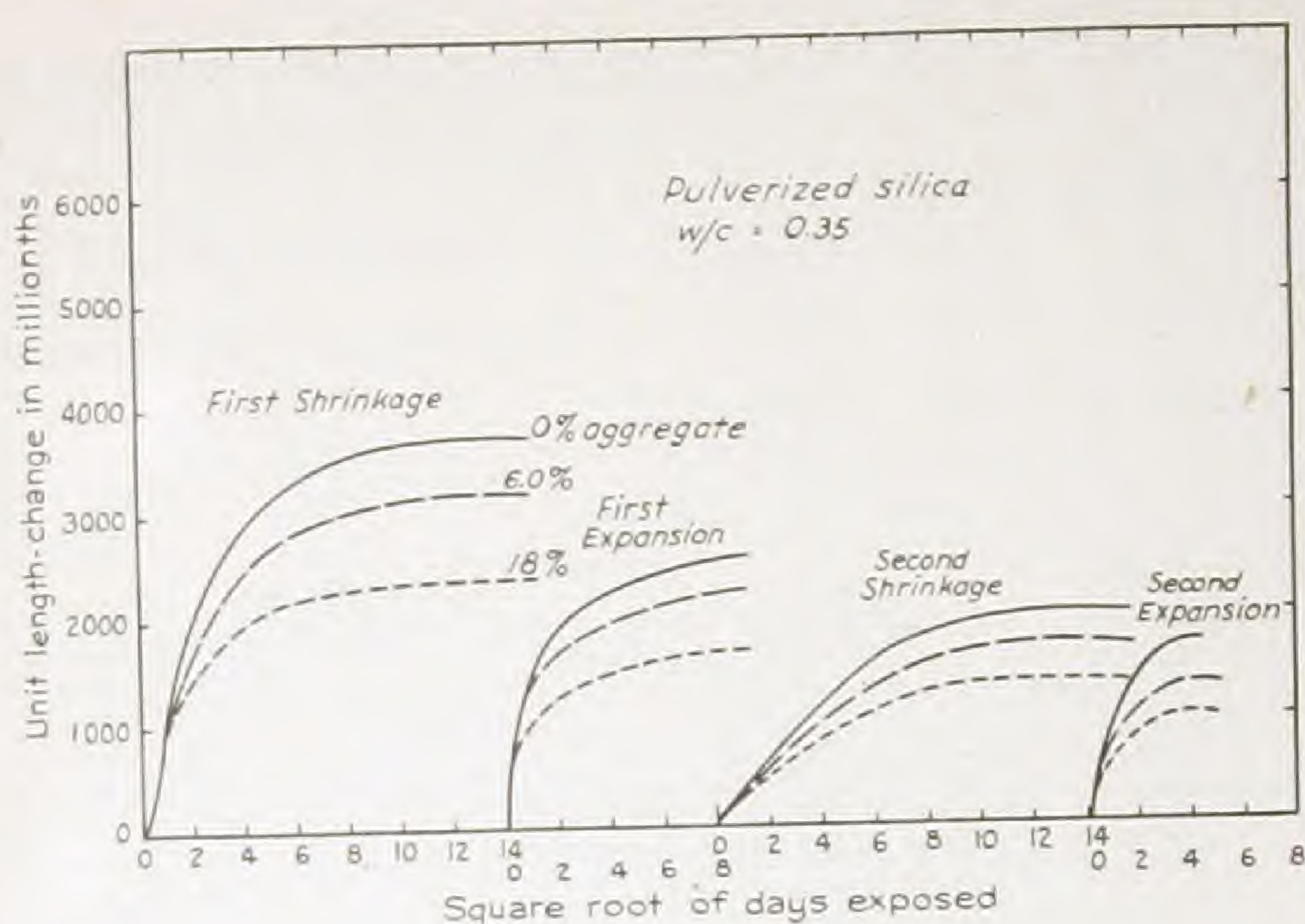
In addition to indicating the validity of the formula for the effect of aggregate on shrinkage, the data give the following indications: (1) First shrinkage is greater than any subsequent expansion or shrinkage. (2) At a given aggregate content first shrinkage is approximately proportional to water-cement ratio. (3) After first shrinkage, subsequent volume changes are approximately independent of water-cement ratio. (4) When shrinkage of specimens of the higher water-cement ratio is plotted against the square root of period of drying, the shapes of the curves for second shrinkage are appreciably different from those for first shrinkage in that they have considerable curvature near the origin.

In general these four indications were either not expected or not expected to the degree indicated by Fig. 1 and 2. Some of the change in behavior might have been due to carbonation during the first drying period, but the major change is believed due to other causes, as will be discussed below.



**Fig. 1—Shrinkage during drying and expansion during wetting for pulverized silica, Ottawa sand, and Elgin sand using high-early-strength cement and  $W/C = 0.50$**





**Fig. 2—Shrinkage during drying and expansion during wetting for pulverized silica, Ottawa sand, and Elgin sand using high-early-strength cement and  $W/C = 0.35$**

## HYPOTHESIS ON GEL STRUCTURE

As a basis for an explanation it is proposed (1) that during first shrinkage some adjacent particles of the cement gel move closer together, whereas others move farther apart, and (2) that, in general, particles that have once made close contact will not return to their original relative positions with subsequent wetting.

No definite picture of gel structure before first shrinkage is required for this analysis except that the gel be slightly altered by the first shrinkage. As water is removed, inter-particle forces will change, necessitating relative movements between particles for equilibrium of individual particles. If these relative movements for each pair of particles are not in proportion to the original distances between their centers, the arrangement will be considered to have changed.

If only discrete colloidal particles were present, shrinkage of hydrated cement might be more nearly like that of soils and show a shrinkage limit,



TABLE 2—FREE SHRINKAGE OF SPECIMENS OF VARIOUS PERCENTAGES OF AGGREGATE

Absolute volume of aggregate per mix volume	Shrinkage in millionths					
	First shrinkage			Second shrinkage		
	Silica	Ottawa sand	Elgin sand	Silica	Ottawa sand	Elgin sand
W/C = 0.50						
0	5870	5870	5870	2180	2180	2180
0.05	4000	5450	5350	1720	2100	2100
0.15	3600	4500	3720	1530	1600	1500
0.30	2200	2850	2700	950	1100	1100
0.50	2000	1700	1650	740	670	640
0.67	—	—	890	—	—	410
W/C = 0.35						
0	3700	3700	3700	2050	2050	2050
0.06	3230	3450	3380	1700	1750	—
0.18	2410	2720	2690	1300	1380	1430
0.34	—	1800	1810	—	1000	1000
0.53	—	940	1080	—	540	620
0.62	—	—	900	—	—	300

*i.e.*, down to some limiting shrinkage, it would show a shrinkage in volume comparable to the volume of water lost. However, in concrete, restraining bodies act from the beginning of drying to reduce shrinkage.<sup>3</sup> The restraining bodies are the aggregates, unhydrated cement grains, and stable micro-crystalline products of hydration. There would, of course, be some differ-

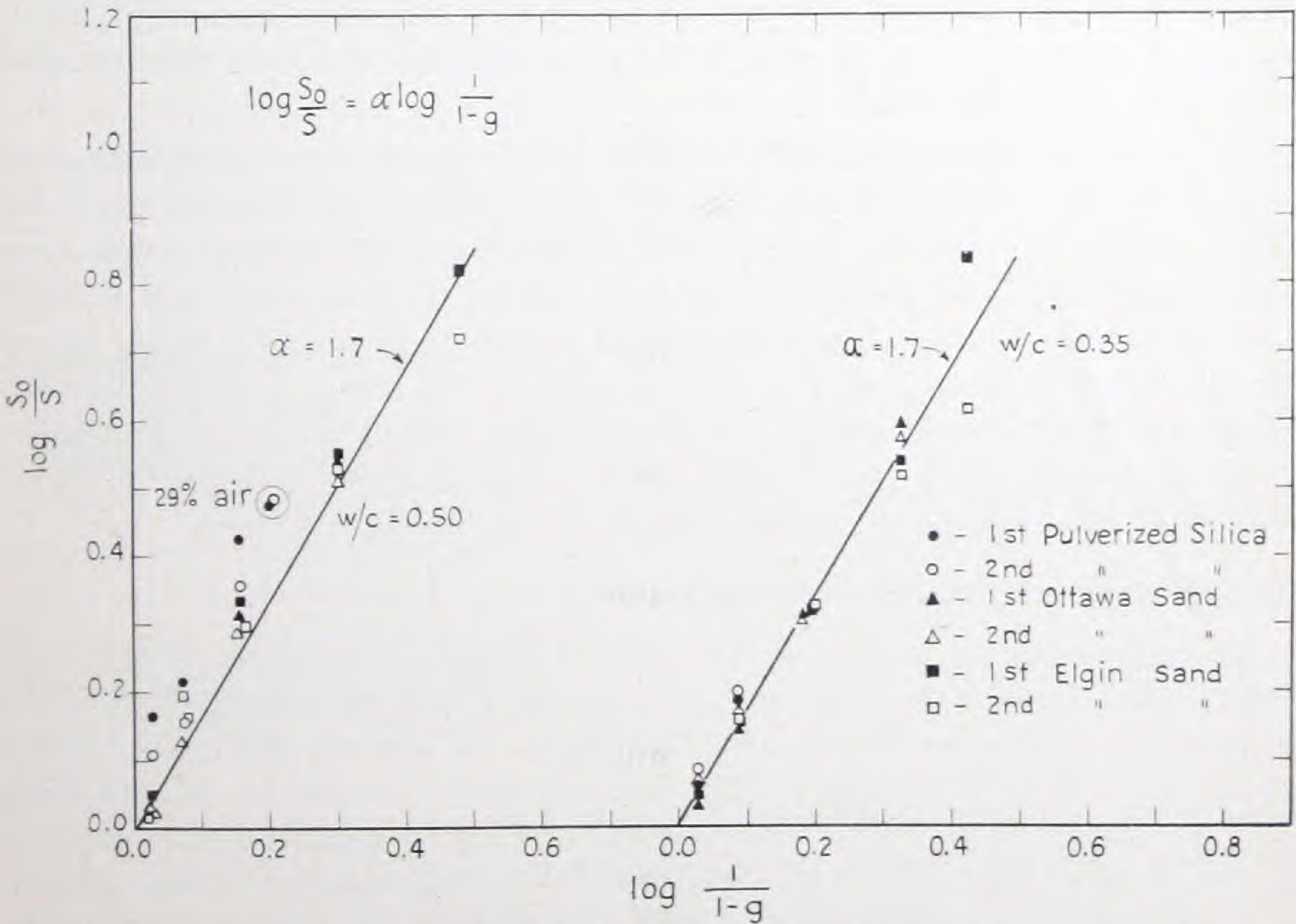


Fig. 3—Effect of aggregate on shrinkage



ence in the intrinsic shrinkage of cement gel and soil because of the bonds between gel particles. Experiments show that concrete shrinkage instead of being about equal to volume of water lost is ordinarily only about 2 to 7 per cent as much. When shrinkage is not equal to loss of water, spaces will form and hydrostatic tension must result. A given gel particle will be under tensile forces tending to pull it toward adjacent particles. These forces may be intense, as is shown below by the relation between hydrostatic tension and relative humidity. Under the conditions for which Kelvin's equation for the curvature of a meniscus in equilibrium with its vapor applies, the intensity of hydrostatic tension of water at room temperature is given by

$$T = -19,600 \log_e h$$

where  $T$  = hydrostatic tension in psi  
 $h$  = relative humidity

For example, if  $h = 0.98$ ,  $T = 392$  psi and if  $h = 0.50$ ,  $T = 13,600$  psi. Kelvin's equation probably is not applicable when radius of curvature is only a few molecular diameters.

Under the action of forces of hydrostatic origin some adjacent particles will be pulled or pushed closer together while other adjacent particles will be pulled farther apart. As two adjacent particles are brought closer together, large compressive forces at the points of closest approach will naturally arise from intermolecular repulsion. These compressive forces should increase with increase in nearby tensile forces so that the particles remain in static equilibrium. As a result of high contact pressures, the particles will probably develop chemical or surface bonds which will tend to prevent future separation of particles, even after hydrostatic tension has been decreased by increase in water content.

The above considerations lead to the conclusion that the first shrinkage alters gel structure so as to change size distribution of spaces between particles. Larger spaces will become larger and smaller ones will become smaller because in general those particles that were closest to each other are brought even closer together and those particles that were separated by larger spaces are pulled farther apart. In this analysis it is not necessary to decide whether the spaces under consideration are the capillary spaces or the much smaller gel pores or both. If the spaces are capillary spaces, then the word "particle" refers to the capillary walls rather than to individual gel particles.

### Application of the hypothesis to test results

The first shrinkage is greater than any subsequent expansion or shrinkage (indication 1, p. 585) because the arrangement of gel particles and groups of gel particles is changed during first shrinkage. At a given aggregate content the extent of first shrinkage should increase with increase in water-cement ratio (indication 2, p. 585). The original spacing of cement grains depends on the water-cement ratio and therefore the average spacing of the gel particles in their first arrangement should also depend on water-cement ratio. More motion during first shrinkage is possible with greater spacing.



After the first shrinkage subsequent volume changes are approximately independent of water-cement ratio (indication 3, p. 585) because after once having been dried the spacing between adjacent gel particles should be more a function of humidity and of the corresponding degree of drying than of original spacing. The gels from mixes of higher water-cement ratio will have a more open structure between agglomerations of particles but not necessarily any greater capacity for changes in volume. This last statement is in accord with the conception of gel structure given by Powers. On the basis of various experiments he concludes that the gel substance has a characteristic spacing of the gel particles.<sup>5,6,7</sup>

The difference in shape of the curves for first and subsequent shrinkages (indication 4, p. 585) is attributed to both the change in distribution of capillary sizes and to the fact that stabilization takes place during first shrinkage but does not occur appreciably during subsequent shrinkages. In any given region in the specimen most of the water in the larger capillaries must be lost before appreciable hydrostatic tension can be developed. During the first shrinkage, before gel structure has become stabilized, appreciable shrinkage can take place with little hydrostatic tension. But after the gel has become stabilized, larger interparticle forces are required to produce comparable shrinkages. The pastes of lower water-cement ratio do not have many large capillaries and therefore, in drying, soon reach the linear portion of the shrinkage versus square-root-of-time relation even though the gel has been stabilized. Moreover, the gels in pastes of low water-cement ratio undergo relatively little structural change during first shrinkage because of the original close particle spacing.

From the above picture it would appear that all volume changes after the first shrinkage should be fairly reversible; however, shrinkage stresses resulting from nonuniform drying or wetting and chemical changes will no doubt cause some change in the structure and therefore prevent complete reversibility.

#### **Explanation of plastic properties of hardened concrete**

As noted by many investigators, concrete has the capacity for a comparatively large amount of creep and the apparent rate of creep for a given stress is relatively large if loads are applied during drying. Although an attempt was made in an earlier paper<sup>8</sup> to show that at least a part of this effect was a natural consequence of nonuniform shrinkage and a nonlinear stress-creep relationship, no satisfactory explanation has been given for the large capacity for creep without failure in tension (cracking) that concrete has while drying as compared to its smaller capacity for creep before or after drying.

By assuming that gel particles change their relative positions, some making closer contacts and others separating during drying, we can understand the large capacity for creep which concrete has while drying. The picture is that each small community of particles will undergo considerable distor-



tional deformation. However, because these deformations are miscellaneously orientated, a region consisting of many such small communities will apparently have no distortion. But if a small stress in a given direction is added, the region will have a resultant distortion which could be of considerable magnitude if high interparticle stresses are also present. After the gel particles have acquired stable positions, the rate of creep will be much less for a given stress and the capacity for creep will be materially reduced, for the action just described cannot take place.

### ACKNOWLEDGMENTS

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**For such discussion of this paper as may develop please see Part 2, December 1956 JOURNAL. In Proceedings V. 52 discussion immediately follows the June 1956 JOURNAL pages.**



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